PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Global Positioning System [and Discussion]

R. J. Anderle, A. L. Allan, N. A. G. Leppard, E. J. Krakiwsky and R. H. Merson

Phil. Trans. R. Soc. Lond. A 1980 **294**, 395-406
doi: 10.1098/rsta.1980.0047

Phil. Trans. R. Soc. Lond. A **294**, 395–406 (1980) 395
Printed in Great Britain

The Global Positioning System

By R. J. Anderle

Naval Surface Weapons Center, Dahlgren, Virginia 22448, *U.S.A.*

The *Navstar* Global Positioning System, proposed for deployment in 1985, will have eight satellites equally spaced in each of three orbit planes at an inclination of 63°. Since the satellites will be in circular, 20000 km (12 h period), orbits and the nodes of the three orbit planes will be equally spaced, at least four satellites will be in view at any location. Range to three of the satellites, computed from the travel time of signal from the satellite to the ground, would give the position of the ground receiver. The measurement to the fourth satellite is required to synchronize the ground station clock with the satellite to provide a sufficiently accurate travel time. In order that the system may be demonstrated in early 1979, six satellites are now being launched into orbits that will provide the operational configuration over the southwestern part of the United States for a few hours each day.

The accuracy of the instantaneous absolute position is expected to be 10 m. The relative position of two stationary receivers could be determined to 1 m accuracy in a few minutes even if the receivers are separated by 1000 km. Relative positions could be determined to better than 10 cm accuracy within a day.

Principle of operation

The Navstar Global Positioning System (G.P.S.) is being developed to provide an observer with a nearly instantaneous determination of his position and velocity in three dimensions. The operational system, proposed for deployment in 1985, would have eight satellites in each of three orbital planes inclined at 63°. The right ascensions of the ascending nodes of the planes would be equally spaced. Since the satellites are equally spaced within each orbit plane at an altitude of about 20000 km (corresponding to an orbit period of 12 h), an observer anywhere in the World would have at least four satellites in view at all times. The satellites broadcast their own positions and the times of emission of coded pulses. If a ground station had a clock synchronized to the satellite clock the range to the satellites would be computed from the difference between the broadcast time of emission of the pulse and the measured time of receipt of the pulse. Ranges to three satellites together with the position of the satellites would yield his position in three dimensions while the range rates would yield his velocity. Since the clocks of the ground observer cannot be maintained in synchrony with the satellite clocks to the required accuracy of a few nanoseconds, measurements are made to a fourth satellite, and the ground station clock is synchronized to the satellite clock by a four-dimensional solution for clock error and three components of position. The 'ranges' computed with the nominal clock readings of the observer before synchronization are termed 'pseudo-ranges', since they are biased by an amount equal to the clock error divided by the speed of light. The operational system is designed to provide the observer with an accuracy of 10 m in position and 0.1 m/s in velocity.

Experimental phase

To provide a test of the concept, three satellites are being launched in each of the two orbit planes at the same altitude and inclination as those proposed for the operational system. The

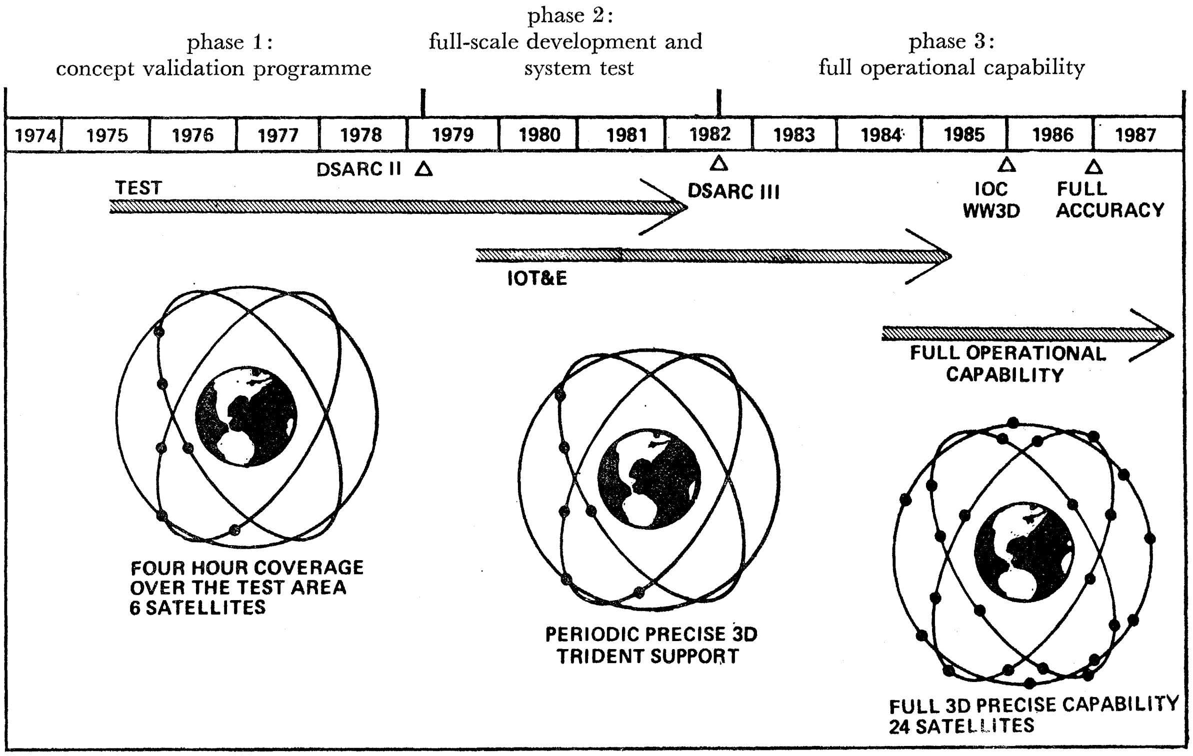
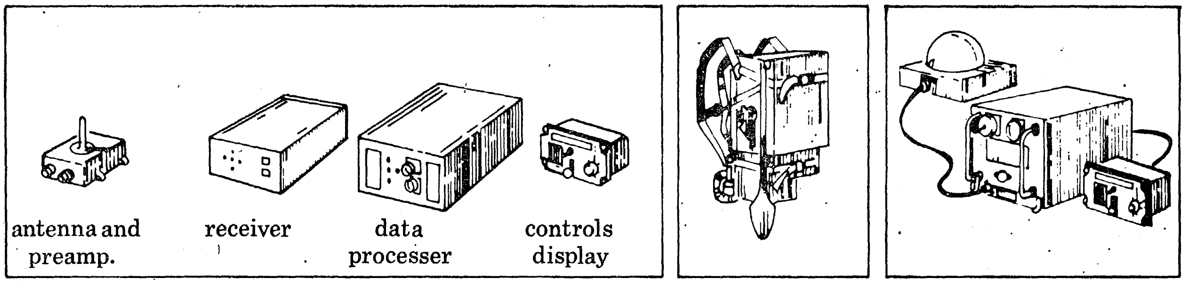


Figure 1. Navstar Global Positioning System schedule.



	X-set HDWE (Magnavox, Texas Inst.)	HI-AJ (Collins)	*Y*-set (Magnavox)	manpack (Magnavox, Texas Inst.)	*Z*-set (Magnavox)
	4 channel, continuous tracking	4 channel, continuous track, i.m.u. aided	1 channel, sequential tracking	1 or 2 channel, sequential tracking	1 channel, sequential tracking
	L_1/L_2, p or c.a.	L_1/L_2, p or c.a.	L_1/L_2, p or c.a.	L_1/L_2, p or c.a.	L_1, c.a. only
	high performance	HI AJ	med. performance, lower cost	lowest size and mass	lowest cost
time to first fix/s	180	150	300	240	300
J/S (precision track)/dB	43	70	40	40	10
size/dm³	159	221	159	16	18
mass/kg	107	152	100	14	13
power/W	860	1000	800	30	75

Figure 2. Phase 1 user equipment.

satellites will be phased in the orbit planes in such a manner that the operational configuration will be achieved for several hours each day over the southwestern part of the United States and at three other geographical locations defined by the orbit configuration. The schedule for implementation of the system is shown in figure 1. During this experimental phase, several types of equipment, illustrated in figure 2, are being tested. The degree of complexity of equipment required by a user depends on his requirements for accuracy and his speed of movement. Where low accuracy is adequate, the observer may observe only one frequency, L_1, at 1575 MHz. This frequency has a course acquisition (c.a.) signal quadraphase modulated with the navigation signal to provide rapid acquisition of the latter signal. For higher accuracy, correlation with the navigation signal on the L_1 and L_2 (1227 MHz) frequencies is required to take advantage both of the precision available from the higher code rate and also of the ionospheric corrections which can be made by measurements on two coherent frequencies. The navigation signal is a pseudo-random noise (p.r.n.) sequence which is biphase modulated onto the carriers at a rate of 10.23 Mbit/s, while the ephemerides and satellite clock data are modulated onto the p.r.n. sequence at a rate of 50 bits/s according to descriptions of the system provided by Martin (1978). A stationary observer could sequence through the satellites and signals with one or two receiver channels devoted to the signals, while a rapidly moving receiver requiring high accuracy should maintain track on four satellites simultaneously. A moving observer can benefit by the use of an inertial measuring unit since a single range measurement is typically made over a time interval of 6 s.

ALTERNATIVES FOR GEODETIC AND GEOPHYSICAL APPLICATIONS

Geodetic and geophysical applications have requirements that differ from those for navigation in a number of ways. On the more stringent side, accuracies of 1 m, or preferably less than 0.1 m, are desired rather than 10 m. On the other hand, the observer can be stationary (with respect to the ground) and can spend hours or days in determining his position, if necessary, rather than seconds. If absolute positions do not provide sufficient accuracy, useful geodetic and geophysical data can be obtained from higher accuracy relative positions of sites, particularly with the use of Navstar satellites, which can be observed by stations which are thousands of kilometres apart. Finally, post-fitted satellite ephemerides can be used to reduce errors in predicted ephemerides and clock times broadcast by the satellite. Relaxing the time to determine position from seconds to one-third of a day allows the use of a different computational approach and/or a different measurement technique for geodetic and geophysical operations from those used in navigation, with attendant advantages and disadvantages. First, the usual pseudo-range measurements made on a single satellite during its passage over the station (which may take 8 h) can be used in position determination in a manner comparable with that used with measurements of Navy Navigation Satellites. These measurements can be treated in two ways, either as uncorrelated differences in range over selected time intervals as is most frequently done with Navy Navigational Satellite data, or as range measurements with a bias. The former procedure reduces the effects of oscillator error while the latter procedure yields a greater precision in the solution. As with observations of lower altitude satellites, observations of a second satellite would greatly increase the geometric strength of a three-dimensional solution. A second measurement technique is to reconstruct the carrier signal, which is suppressed, and count carrier cycles to provide data comparable to Navy Navigation Satellite data. These data

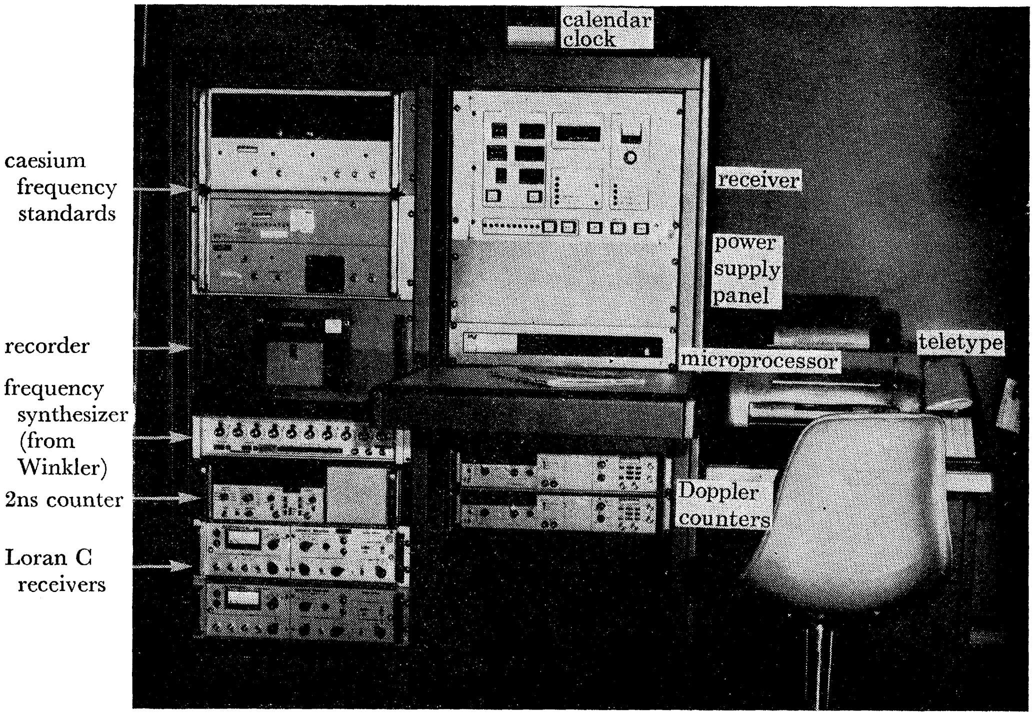


Figure 3. Navstar geodetic receiver and test equipment.

can also be treated as uncorrelated range differences or as biased ranges. Experimental equipment developed by the Naval Surface Weapons Center to make these measurements is shown in figure 3 with some associated test equipment. Measurements of the carrier signal can be made with considerably greater precision than measurements of the code signal. Measurements of either carrier or code throughout a pass of two satellites, rather than of code on four satellites, can reduce the complexity of the ground equipment, but greatly increases the requirement for an ultra-stable oscillator.

Error budget for navigation applications

The error budget for the projection of the various error sources on a typical range vector from a ground station to a satellite is shown in table 1. When this 'user equivalent range error' is multiplied by about three, the result is the expected accuracy in the determination of a ground position. The multiplicative factor, which is slightly higher for the vertical than for the horizontal component of position, is termed the 'geometric dilution of precision' (GDOP). If it were not necessary to solve for the error in the ground station clock, the GDOP would be near

Table 1. G.P.S. error budget: user equivalent range error (metres)

	phase I	phase II
ephemeris	3.9	1.6
atmospheric delay	2.6	2.6
group delay	2.6	1.0
receiver noise	1.6	1.6
multipath	1.3	1.3
r.s.s.	5.9	3.9

unity for the Navstar constellation of satellites. The errors in the satellite ephemeris and clock would be significantly smaller if a post-fitted ephemeris could be used for the calculations rather than the predicted ephemeris broadcast by the satellite.

Ephemeris accuracy

The expected errors in the satellite ephemeris computed from 6 days of observations from four ground stations is shown in figure 4. The largest error components are those parallel to the direction of motion of the satellite (tangential) and those normal to the orbit plane. However, even at the worst elevation angles the projection of these error components is reduced by a factor of four in their effect on computed station position owing to the height of the satellite, while the projection of these components is zero for overhead passes. Note that the error components are below 2 m in the normal and tangential direction and below 0.2 m radially during the central day of the 6 day fit. The fitting procedure used to produce the navigation message injected into the satellite would tend to place this minimum near the last observation in order to reduce the error in the predicted ephemeris.

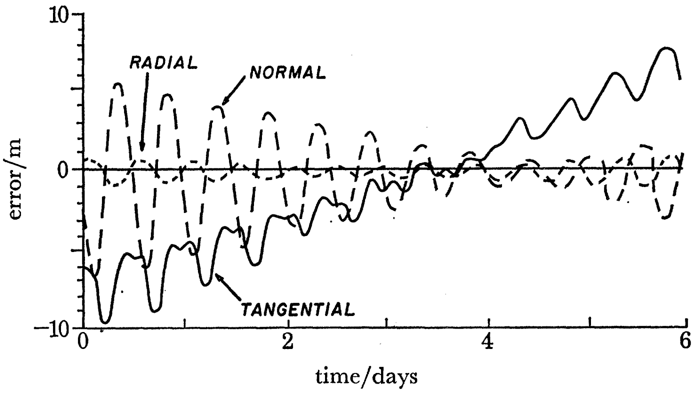


Figure 4. Deterministic orbit errors.

Accuracy of absolute geodetic positions

The effect of errors in the predicted satellite ephemeris on the computed position of a ground receiver is shown in figure 5 (Hill 1978). The position of the receiver is obtained to 1 m accuracy very rapidly, but as new satellites come into view, each having independent errors in radial, crosstrack, and tangential position of 2, 6 and 10 m respectively, the sequential processor used in this simulation allows the computed position of the receiver to vary between 60 and 90 cm. Other simulations have shown that even if fitted satellite ephemerides rather than predicted ephemerides are used, the accuracy of computed absolute station positions is limited to this level of accuracy owing to uncertainties in computed position of the Earth's spin axis with respect to the geodetic system. While improvements in equipment, refinements in computational procedures, or development of very long baseline interferometric systems for rapid determination of pole position and Earth's rotation may ultimately ameliorate this limitation on accuracy of absolute station position, it is more prudent to compute relative station positions to achieve

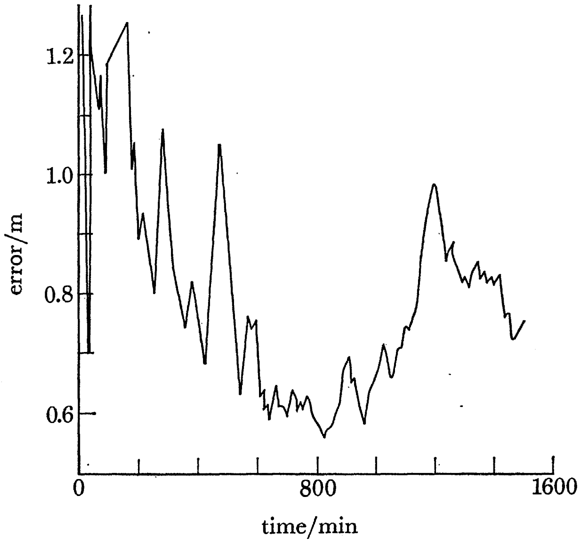


FIGURE 5. Position error.

sub-decimetre accuracy in position. Owing to the great altitude of the satellite, such accuracies can be achieved over distances as great as 1000 km.

EFFECT OF EPHEMERIS ERRORS ON RELATIVE STATION POSITIONS

Let us consider the error in the relative ranges from two stations to the satellite resulting from an error in the predicted position of the satellite. An orbit error normal to the plane containing the range vectors from the two stations is completely negligible because it is normal to at least a 20000 km distance to the satellite. An error in the plane of the two range vectors and normal to the base line between the stations projects on the relative range vectors as shown in figure 6.

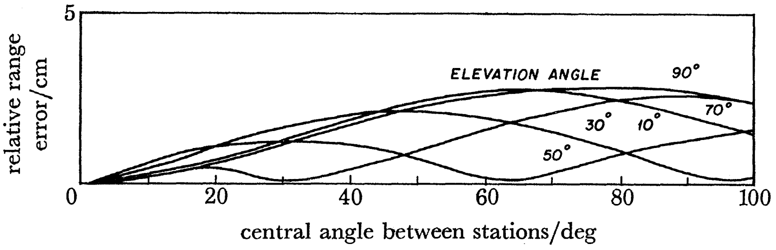


FIGURE 6. Difference in range error at two stations due to 1 m radial orbit error.

The error rises from zero for co-located stations to no more than 3 % of the orbit error for widely separated stations. When the plane is vertical, the relative range error for this component of the ephemeris error would be below 0.6 cm for a fitted ephemeris or below 6 cm for a predicted ephemeris. For planes containing the range vectors that are inclined from the vertical, the relative range vectors would include a component of the orbit error normal to the orbit plane, which is a factor of ten larger than these values for a fitted trajectory and a factor of three larger for a predicted ephemeris; but the error is still null for certain positions between the stations, as shown on figure 6. The component of ephemeris error that has the largest projection on the relative range error is that parallel to the direction of travel of the satellite. To maintain the relative range error below 3 % of this component of the ephemeris error, stations would have to be less than 1000 km apart as shown in figure 7. To summarize, ephemeris errors are not expected to contribute more than a few centimetres to the uncertainty in the relative positions of stations which are separated by less than 1000 km.

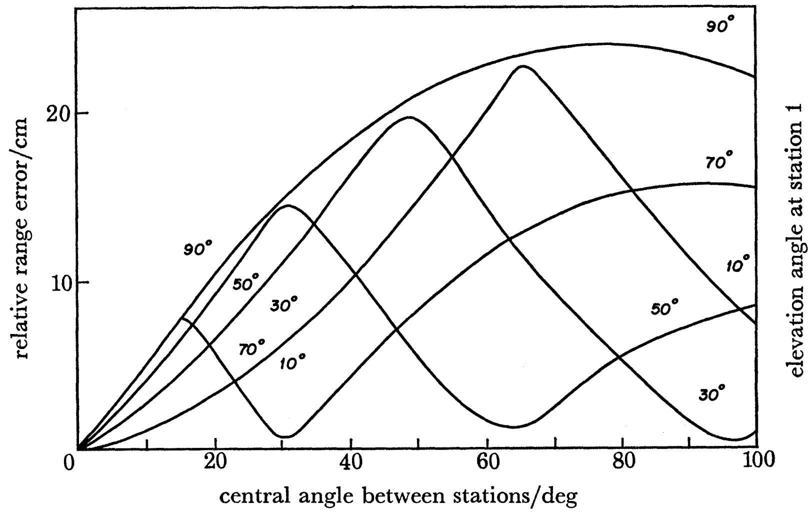


Figure 7. Difference in range error at two stations due to 1 m orbit error in plane of stations normal to orbit radius.

The Doppler approach

As indicated earlier, station positions can be computed on the basis of range differences to a satellite with respect to time during the pass of one or more satellites over a station, which is the technique currently used with observations of Navy Navigational Satellites. The G.P.S.

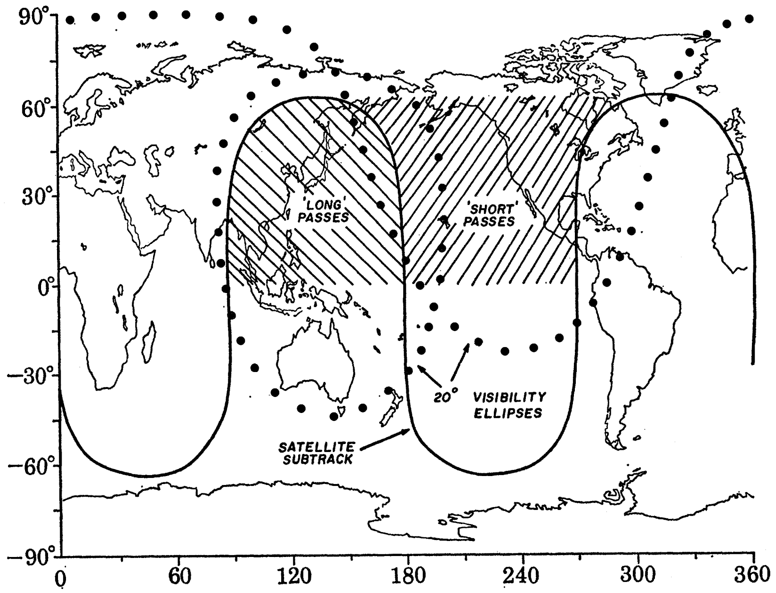


Figure 8. Dependence of pass length on relative longitudes of station and satellite.

satellites, however, can enter and leave the visibility circle about a station twice within an orbit period as shown on the right of figure 8. In simulations, such data are treated as two short passes. At other locations, the satellite remains within view of the station for a substantial portion of the orbit period, resulting in a long pass as shown on the left of the figure. The precision of a two-dimensional determination of station position was computed for such range difference observations to a single satellite. The computations were performed in two ways: in one solution, the range differences over successive time intervals were treated as uncorrelated, which is the most frequent treatment accorded observations of the Navy Navigational Satellites. In the second solution, the observations were treated as correlated, which is equivalent to considering the observations to be of range with an arbitrary bias throughout the pass. For the uncorrelated range difference solution, the parameters of the solution for each pass were two

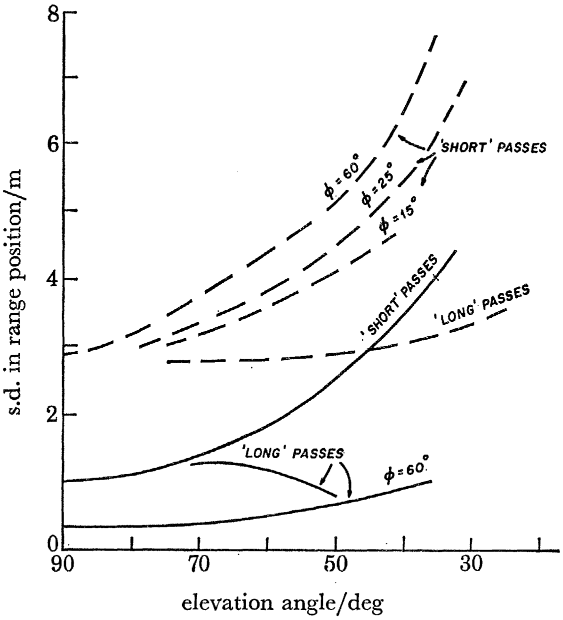


FIGURE 9. Standard error in range per 1 m random error in phase measurement at 15 min intervals. – – –, Uncorrelated range difference; ——, biased range; ϕ = latitude.

components of station position, a frequency bias and a refraction bias with a 10 % uncertainty assigned to the nominal refraction model. The two components of station position were in the plane containing the range vector to the satellite and the velocity vector of the satellite at the time of closest approach of the satellite to the station. One component was in the direction of the range vector and one component was parallel to the velocity vector (termed the 'tangential' position component). For the biased range solution, the range bias was added to the parameter set. The precision of the solution for the two data classes is shown in figures 9 and 10 for the range and tangential position components respectively. It will be seen that the precision worsens very rapidly with decreasing elevation angles for short passes. For the biased range data representation, the precision of solution is about equal to the precision of the data, while for uncorrelated range difference representation, the precision of solution is three times worse than the precision of the data. This is a pessimistic view of the biased range representation, since increased precision can be achieved with an increased data rate. On the other hand, an increased

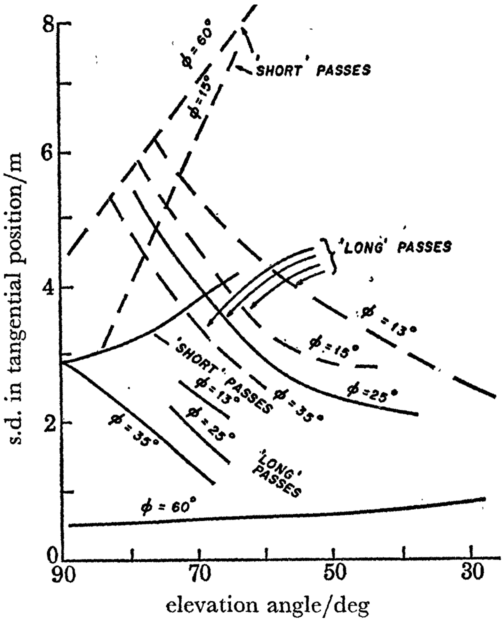


FIGURE 10. Standard error in tangential position per 1 m random error in phase measurement at 15 min intervals. Data representation as in figure 9.

data rate for the uncorrelated range difference representation would actually worsen the precision of the data. The figures imply that phase measurements, which can be made to centimetre precision, would yield precisions of 1 cm in station position. Range measurements made on this pseudo-random noise signal, which typically have about a 1.5 m precision for a 6 s measurement interval, would yield about 12 cm precision when aggregated to the 15 min intervals assumed in the computations that produced figures 9 and 10, corresponding to a 12 cm precision in station position.

The term 'precision' has been emphasized in this section because the accuracy of the solution is highly dependent on the stability of the oscillator in the ground station. Simulations of the

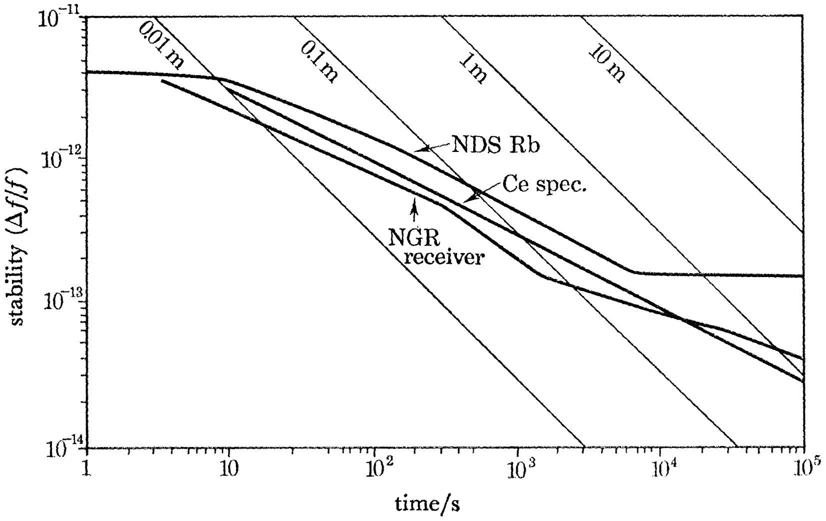


FIGURE 11. Allan variance.

effects of oscillator instability on these solutions have not been completed, but an indication of this importance can be seen in figure 11 which shows the frequency stability for several oscillators as the ordinate as a function of averaging time as abscissa. The integrated effect of the instability can be found by interpolating between the diagonal lines which are labelled in metres across the top of the graph. Thus, the instability in the oscillator used in the Navstar Geodetic Receiver (N.G.R.) would produce about a 50 cm integrated error over a period of about 8 h. Since oscillator frequency is a parameter of the pass, the critical period is something shorter than the total pass length. Fell (personal communication) found that residuals for a frequency solution to 8 h of range data for an oscillator similar to that shown for the N.G.R. receiver have a root mean square error of about 10 cm (figure 12). However, this error would be systematic within a pass, so the curves for the effects of random error given earlier cannot be used to assess the effects on position.

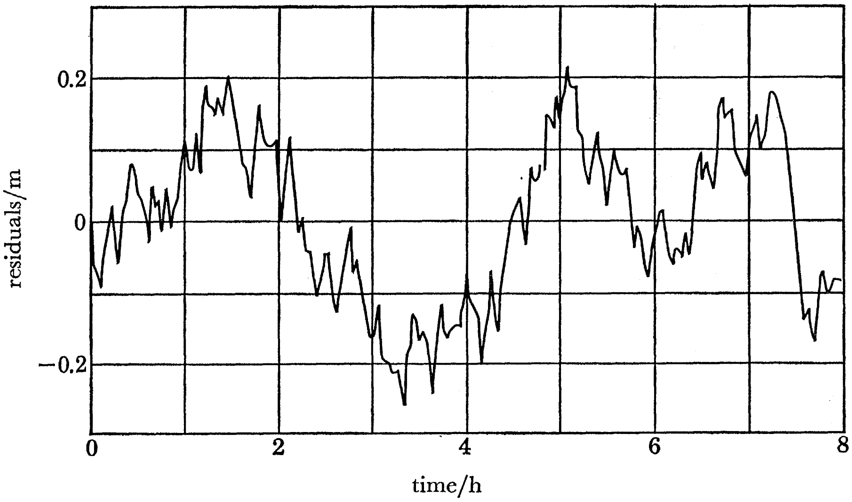


Figure 12. Residuals to a linear fit.

Summary

The G.P.S. satellite system will provide absolute geodetic positions to 1 m accuracy with a few hours of observation, and it also has the potential of yielding the relative position of stations to sub-decimetre accuracy by use of a variety of data types and data analysis techniques. The effects of random observational error on position for some techniques discussed in this report are only a few centimetres. Each of these techniques is subject to the effects of additional systematic errors. Effects of ephemeris errors on position are just a few centimetres for fitted ephemerides for stations separated by less than 1000 km. Each is also subject to negligible second order ionospheric refraction errors and tropospheric refraction errors which will reach a few centimetres (Anderle 1978) in their effects on station position. The systems are sensitive in different degrees to oscillator stability, equipment biases and multipath effects. Further simulations are being conducted to evaluate the effects of oscillator stability on position calculations based on Doppler data; equipment biases must be calibrated and evaluated in field tests and multipath effects must be evaluated in field tests with different antenna configurations.

REFERENCES (Anderle)

Anderle, R. J. 1978 Geodetic Applications of the Navstar global Positioning System. Presented at Second International Symposium on Problems Related to the Redefinition of North American Geodetic Networks, Washington D.C.

Hill, R. W. 1978 Applications of the Navstar Global Positioning System to marine geodesy. Presented at International Symposium on Interactions of Marine Geodesy and Ocean Dynamics, Miami, Florida.

Martin, C. F. 1978 Impact of navigation and geodesy compatibility on geodetic measurements. Presented at International Symposium on Geodetic Measurement and Computations, Zaria, Nigeria.

Discussion

A. L. ALLAN (*University College London, U.K.*). Is there any plan to have satellites ranging to each other?

R. J. ANDERLE. This approach was considered early in the programme, but I am not aware of any current plans for receivers aboard the satellite.

A. L. ALLAN. Can the satellite orbits be corrected later? If so, what will this do to the ephemerides?

R. J. ANDERLE. Yes, the satellites contain thrusters to permit adjustment of the phases of the satellites in their orbits. In addition, the energy built up in momentum wheels used for orientation stabilization is dissipated by jet torques which are nominally balanced, although we must consider a possible imbalance. However, the momentum dumps are normally one per week and the intervals between phase adjustments will probably be measured in months, which will present no problem.

N. A. G. LEPPARD (*Directorate of Military Survey, Feltham, Middlesex, U.K.*). For navigation control of Navstar G.P.S. some four monitor stations will be employed in the Pacific area. For geodetic purposes, do you envisage that additional monitors will be required?

R. J. ANDERLE. Even for the navigation control there may be a need for additional monitoring stations, and more will be required for the most precise geodetic applications.

E. J. KRAKIWSKY (*University of New Brunswick, Canada*). What is the underlying geometrical configuration and mathematical model used in obtaining the simulated positional errors?

R. J. ANDERLE. The position errors are based on observations made on one satellite by one station at 15 min intervals from the time the satellite enters a 70° zenith cone about the station until it leaves the visibility cone. Random errors of observations are propagated through the normal equations to obtain standard errors in two components of station position, refraction bias (with 10 % uncertainty for *a priori* refraction) and frequency bias. For range observations, a range bias parameter was included in the normal equations.

R. H. MERSON (*Royal Aircraft Establishment, Farnborough, U.K.*). In view of the strong resonance effect on a 12 h satellite of the 3:2, 5:2, 7:2, ..., terms in the geopotential, we have analysed 12 days of NTS-2 data to compare the observed resonance effect with that given by the GEM 10 field. It seems possible that there is a 5 % discrepancy. Has this strong resonance been considered as a subject for watching during the next stage of the Navstar project?

R. J. ANDERLE. Effects of differences between independent solutions for the gravity field have been found to be small compared with the effects of uncertainties in the solar radiation model.

Of course the differences could have been small fortuitously; in any case, solution for an improved gravity model is planned.

R. H. Merson. The range difference method of analysing Navstar data would seem to be equivalent to passing the data through a weak high-pass filter. If the caesium clock spectrum can be obtained, would it not be appropriate to determine the optimum filter to match the data to the system geometry?

R. J. Anderle. I agree that the current procedure, which was adopted as an expedient way of accounting for oscillator instability in the statistics of the solution, is sub-optimal. Two alternatives are being pursued based on the measured Allan variance of the oscillator. A sequential processor has been developed in which the oscillator error is modelled by a Markov process, and a batch processor is being developed which uses a non-diagonal weight matrix.

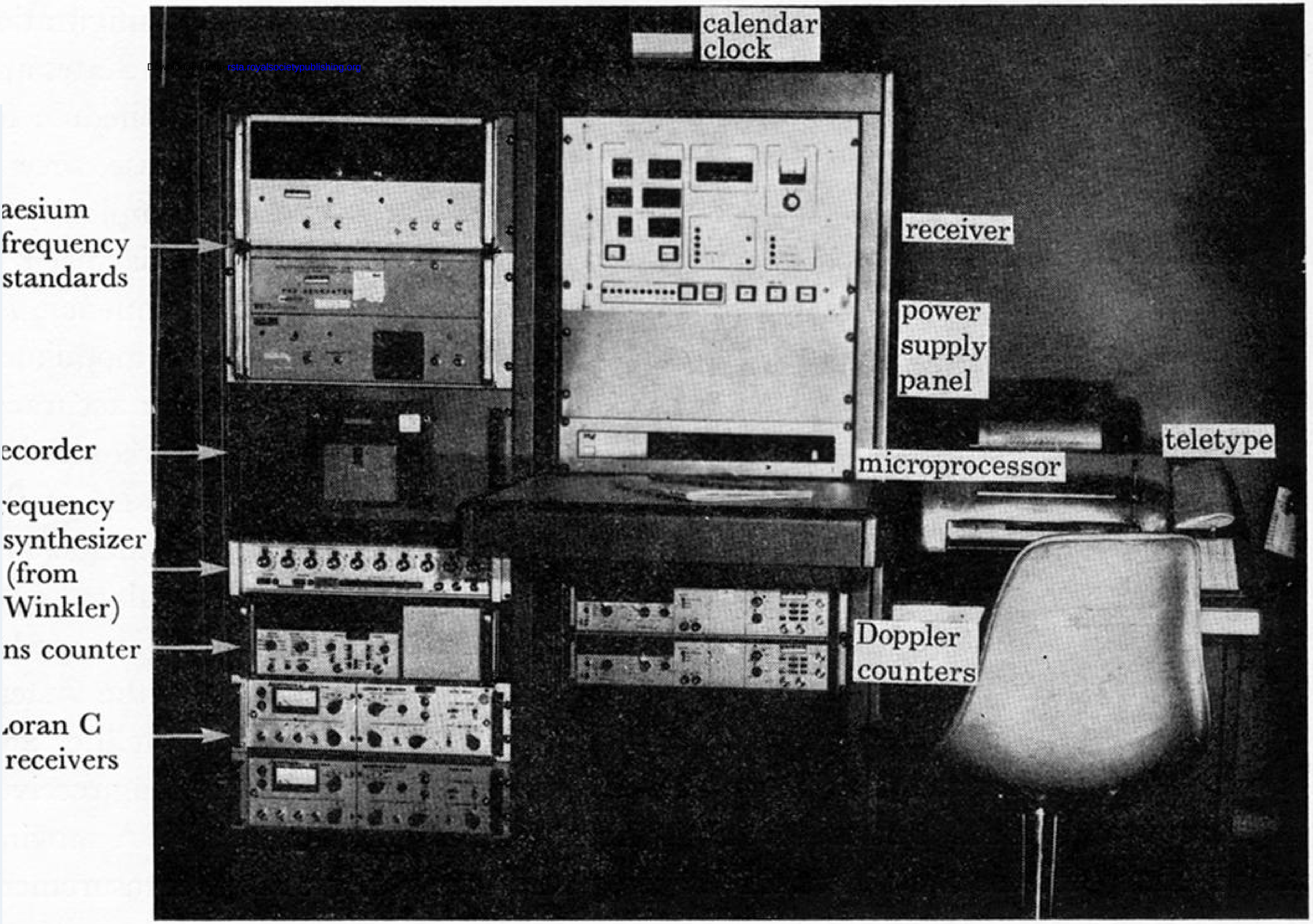


Figure 3. Navstar geodetic receiver and test equipment.